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Five Early Theories in the Mediaeval Insolubilia-*Literature*

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Mediaeval logicians, from at least the second half of the twelfth century, devoted much time and effort to semantic paradoxes of the Liar type ("This sentence is false" or "This sentence is not true"). Such paradoxes were called "*insolubilia*"—"insolubles".[1] Despite the term most authors were convinced that it was in fact possible to "solve" such paradoxes; the solution was just very difficult to find.[2] It is not surprising, therefore, that we see a wide variety of suggested solutions in the mediaeval literature on this problem.[3] It is purpose of this paper to examine and survey such solutions during the early history of that mediaeval literature.

I

The earliest known mediaeval discussions of paradoxes like the Liar took place in the middle or late twelfth century. In his *Ars disserendi*, for

[1] This claim needs some qualification. In fact few authors attempted to give a rigorous definition of an insoluble, and those who did try gave definitions that were either too broad or too narrow to fit their intentions or even their practice. Some authors considered under the heading of "*insolubilia*" various paradoxes or puzzles that bear little relation to the Liar. See, for example, John Buridan's *Sophismata*, Ca. 8, sophisms 1-6, in *John Buridan On Self-Reference: Chapter Eight of Buridan's 'Sophismata', with a Translation, an Introduction, and a philosophical Commentary*, G. E. Hughes, ed. & tr., (Cambridge: Cambridge University Press, 1982), pp. 38-61, with a commentary on pp. 141-161. In the paperback edition, which includes the translation and commentary but omits the Latin text, the corresponding pages are pp. 34-45 and pp. 80-100. For an edition of the entire *Sophismata*, see *Johannes Buridanus, Sophismata*, T. K. Scott, ed., (Grammatica Speculativa, vol. 1, Stuttgart-Bad Cannstatt 1977), translated by T. K. Scott in *John Buridan: Sophisms on Meaning and Truth*, (Century Philosophy Sourcebooks, New York 1966). But notwithstanding these other kinds of paradoxes, it was variations on the Liar paradox that appear to have been the central concern of the *insolubilia*-literature.

[2] See the discussion of this attitude in my introduction to William Heytesbury, *On "Insoluble" Sentences: Chapter One of His Rules for Solving Sophisms*, Paul Vincent Spade, tr., (Mediaeval Sources in Translation, vol. 21, Toronto 1979), pp. 7-11. Heytesbury himself is the only mediaeval author I know who did not share this optimistic view, and even he was not altogether consistent about the matter. See *ibid.*

[3] For a survey of this mediaeval literature, see Francesco Bottin, *Le antinomie semantiche nella logica medievale*, (Padua 1976); and Paul Vincent Spade, *The Mediaeval Liar: A Catalogue of the Insolubilia-Literature*, (Subsidia Mediaevalia, vol. 5, Toronto 1975).

instance, written in 1132,[4] Adam of Balsham asks "Whether he lies who says nothing but that he lies,"[5] and also "Whether he says the truth who says nothing but that he lies".[6] These are the earliest known mediaeval formulations of the Liar-sentence. But Adam presents them as examples of an altogether different point. He says nothing whatever to indicate that he was aware of the very special problems they pose, that they were current topics of philosophical discussion in his day, or how one might go about trying to answer those questions.[7]

Nevertheless, there is other evidence to indicate that the special problems associated with such sentences were realized and discussed by the second half of the twelfth century if not before.[8] The anonymous *Dialectica Monacensis*, for instance, includes the following remark in its account of the Aristotelian fallacy *secundum quid et simpliciter*: "But how this fallacy arises in uttering the insoluble 'I am saying a falsehood', that is a matter discussed in the treatise on insolubles."[9] This passage, from some time in the second half of the twelfth century,[10] contains the first known occurrence of the word '*in-*

[4] On the date, see *Adam Balsamiensis Parvipontani Ars Disserendi* (*Dialectica Alexandri*), Lorenzo Minio-Paluello, ed., (Twelfth Century Logic: Texts and Studies, vol. 1, Roma 1956), p. xxi. This date must be used with some caution. The *Ars disserendi* had at least two recensions. The first, which appears to go back to Adam himself in 1132, is preserved for only part of the text. The second recension is quite different and presumably later. The passages from "Adam" quoted below come from this second recension; there is no guarantee that they were present in the original. There is manuscript evidence linking the text of the *Ars disserendi* to a certain "Alexander", who is perhaps to be identified with Alexander Neckham, a later member of Adam's school at the Petit Pont. On all these matter, see *ibid.*, pp. xiii-xxii. Neckham in any case has a role in our story. See below.

[5] *Ibid.*, p. 86.21-22: "An mentiatur qui nichil nisi se mentiri dicit." (All translations in this paper are my own.)

[6] *Ibid.*, p. 107.26-27: "An vere dicat qui nichil nisi se mentiri dicit." See also *ibid.*, p. 107.25-26: "An vere enuntiet qui se mentiri dicit."

[7] See the discussion of these passages in Paul Vincent Spade, *The Origins of the Mediaeval* Insolubilia-*Literature*, in: Franciscan Studies, 33 (1973), pp. 292-309 at pp. 294-295.

[8] I did not take sufficient account of this evidenc in my *The Origins of the Mediaeval* Insolubilia-*Literature*, where I maintained that the earliest explicit discussions of *insolubilia* could not be dated with certainty before the beginning of the thirteenth century.

[9] See L. M. De Rijk, *Logica Modernorum: A Contribution to the History of Early Terminist Logic*, vol. II: *The Origin and Early Development of the Theory of Supposition*, (Assen 1967), Part 2, p. 594.30-31: "Qualiter autem fiat hec fallacia in dictione illius insolubilis: '*ego dico falsum*', hoc habetur in tractatu de insolubilibus." The "treatise on insolubles" is not contained in the *Dialectica Monacensis* as we have it today. See *ibid.*, n, 1.

[10] On the date, see *ibid.*, Part 1, pp. 410-414.

solubile' as a technical term for this kind of paradoxical sentence,[11] and shows that already separate treatises were devoted to the topic. By itself, however, the passage provides no evidence that the special, antinomial nature of such sentences was understood.

Such evidence is unequivocally to be found, however, in another late twelfth century text. Alexander Neckham, in his *De naturis rerum*, writes:[12]

> Again, if Socrates says he lies, and says nothing else, he says some proposition. Therefore, either a true one or a false one. Therefore, if Socrates says only that he lies, he says what is true or what is false. But if Socrates says only the proposition that Socrates lies, and he says what is true, then it is true that Socrates lies. And if it is true that Socrates lies, Socrates says what is false. Therefore, if Socrates says only the proposition that Socrates lies, and he says what is true, he says what is false. But if Socrates says only the proposition that Socrates lies, and he says what is false, then it is false that Socrates says what it false. And if it is false that Socrates says what is false, Socrates does not say what is false. But if Socrates says only that he lies, he says either what is true or what is false. Therefore, if Socrates says he lies, he says what is true. Therefore, if Socrates says only that he lies, and he says what is false, then he says what is true. But if Socrates says only that he lies, he says what is true or false. Therefore, if Socrates says only that he lies, he says what is true *and* says what is false.

This text shows a clear understanding of the problem sometime in the late twelfth century.[13] On the other hand, Neckham gives these

[11] On the claim that such a use of the term may be found as early as 1159 in John of Salisbury's *Metalogicon*, see Spade, *The Origins of the Mediaeval* Insolubilia-*Literature*, pp. 295-296.

[12] Alexander Neckham, *De naturis rerum libri duo, with the Poem of the Same Author, De laudibus divinae sapientiae*, Thomas Wright, ed., (Rerum Britannicarum Medii Aevi Scriptores, or Chronicles and Memorials of Great Britain and Ireland during the Middle Ages (Rolls Series), no. 34; London 1863; reprinted Kraus Reprint, 1967), p. 289: "Item, si Sortes dicit se mentiri, et nihil aliud dicit, dicit aliquod enuntiabile, ergo verum vel falsum; ergo si Sortes dicit tantum se mentiri, dicit verum vel falsum, sed si Sortes dicit tantum id quod est Sortem mentiri, et dicit verum, verum est Sortem mentiri, et si verum est Sortem mentiri, Sortes dicit falsum; ergo si Sortes dicit tantum id quod est Sortem mentiri, et dicit verum, dicit falsum; sed si Sortes dicit tantum id quod est Sortem mentiri, et dicit falsum, falsum est Sortem dicere falsum, et si falsum est Sortem dicere falsum, Sortes non dicit falsum; sed si Sortes dicit tantum se mentiri, dicit verum vel falsum; ergo si Sortes dicit se mentiri, dicit verum; ergo si Sortes dicit tantum se mentiri, et dicit falsum, dicit verum; sed si Sortes dicit tantum se mentiri, dict verum vel falsum; ergo si Sortes dicit tantum se mentiri, dicit verum et dicit falsum." Alexander goes on to give other versions of the paradox, *ibid.*, pp. 289-290, but this suffices for my present point. All these passages are reproduced in De Rijk, *op. cit.*, Part 1, pp. 290-291.

[13] The date of the *De naturis rerum* in uncertain, although it was apparently well known at the end of the century. See Wright's introduction to Neckham, *op. cit*, pp. xiii-xiv. Neckham was born in 1157. (*Ibid.*, p. ix.) In view of the above discussion of Adam of Balsham, it is perhaps significant that Neckham had studied at the Petit Pont, at the

versions of the paradox only as examples of the kinds of "vanities" one deals with in dialectic. He offers no indication of how one might—or did—try to "solve" them.

It is not until the turn of the century that we find still extant discussions that not only show an explicit awareness of the antinomial features of *insolubilia*, but also offer or refer to more or less identifiable attempts to resolve them.[14] From that time on, we can distinguish three main stages in the mediaeval development of this literature:[15]

(1) An early or seminal stage, from about 1200 to roughly 1320.
(2) A short stage of intense and sophisticated activity, centered mainly at Merton College, Oxford, from about 1320 until about 1350.
(3) A late stage, from around 1350 to the end of the Middle Ages,[16] a period that,

school founded by Adam. (*Ibid.*, p. x.) It is perhaps also significant that both Adam and Neckham were Englishmen. De Rijk (*op. cit.*, Part 1, p. 290) remarks that "all the theses discussed by Alexander" in the text containing the passage cited above "are found in the Ars Meliduna," which he dates between 1154 and 1180 (*ibid.*, pp. 280-281) and attributes to a Frenchman (*ibid.*, pp. 279-280). But I have been unable to locate any unequivocal references to *insolubilia* in the description and quotations from this work published by De Rijk, *ibid.*, Chs. 6-10, pp. 264-390.

[14] The earliest extant treatise of this kind is discussed and edited in L. M. De Rijk, *Some Notes on the Mediaeval Tract* De insolubilibus, *with the Edition of a Tract Dating from the End of the Twelfth Century*, in: Vivarium 4 (1966), pp. 83-115.

[15] See also the discussion of these three stages in Paul Vincent Spade, *Insolubilia*, in: *The Cambridge History of Later Medieval Philosophy*, Norman Kretzmann, *et. al.*, eds., (Cambridge 1982), Ch. 12, pp. 246-253. For another account of some of the differences between the first two stages, see Francesco Bottin, *The Mertonians' Metalinguistic Sciences and the* Insolubilia, in: P. Osmund Lewry, ed., *The Rise of British Logic: Acts of the Sixth European Symposium on Medieval Logic and Semantics, Balliol College, Oxford, 19-24 June 1983*, (Papers in Mediaeval Studies, Vol. 7, Toronto [1985]), pp. 235-248.

[16] For the post-mediaeval history of *insolubilia*, see E. J. Ashworth, *Language and Logic in the Post-Medieval Period*, (Synthese Historical Library, vol. 12, Dordrecht 1974), pp. 101-117; and E. J. Ashworth, *The Treatment of Semantic Paradoxes from 1400 to 1700*, in: Notre Dame Journal of Formal Logic, 13 (1972), pp. 34-52.

[17] There were some noteworthy exceptions: for instance, the views of John Wyclif and Peter of Ailly. On Wyclif's view as contained in his *Logicae continuatio*, see Spade, *The Mediaeval Liar*, item xliii, pp. 74-76. Wyclif also wrote an independent *Summa insolubilium*, attributed in some manuscripts to John Tarteys. (The work I attributed to Tarteys *ibid.*, item xl, p. 70, is in fact Wyclif's *Summa insolubilium.*) See John Wyclif, *Summa insolubilium*, Paul Vincent Spade and Gordon A. Wilson, eds., (Binghamton, NY: Medieval & Renaissance Texts & Studies, forthcoming). The treatise described as item xliv, and tentatively attributed to Wyclif, in *The Mediaeval Liar*, pp. 76-77, is in fact not by Wyclif at all (although it is influenced by him), but rather by Robert Alyngton. See L. M. De Rijk, *Logica Oxoniensis: An Attempt to Reconstruct a Fifteenth Century Oxford Manual of Logic*, in; Medioevo, 3 (1977), pp. 121-164 at pp. 139-140. On Peter of Ailly's view, see Paul Vincent Spade, *Peter of Ailly: Concepts and Insolubles, An Annotated Translation*, (Synthese Historical Library, vol. 19, Dordrecht 1980). Peter's views appear to be derived in part from those of Gregory of Rimini and Marsilius of Inghen in ways that need to be investigated more fully. See *ibid.*, introduction, pp. 6-7.

as far as *insolubilia* are concerned, was devoted mainly to the careful elaboration and refinement of views originally developed during the second stage.[17]

The turning point between the first and the second of these stages was the *Insolubilia* of Thomas Bradwardine, written probably between 1320 and 1324.[18] The topic of this paper, therefore, is the *insolubilia*-literature from the turn of the twelfth/thirteenth century to the time of Bradwardine.

In my earlier paper "The Origins of the Mediaeval *Insolubilia*-Literature,"[19] I investigated possible sources for the mediaeval discussion of these paradoxes. The results of that paper need not be rehearsed here, except to note that the mediaeval literature seems to have taken its inspiration from a passage in Aristotle's *Sophistici elenchi* that perhaps suggests but by no means explicitly formulates the antinomy. In that passage, part of the treatment of the fallacy *secundum quid et simpliciter*, Aristotle mentions the case of a man who swears that he will commit perjury and then does so. Is that man a perjurer or not? Absolutely (*simpliciter*), Aristotle says, he is; it is only in a certain respect (*secundum quid*), namely with respect to the particular oath that he would commit perjury, that he keeps his oath.[20] Aristotle then goes on

[18] On the date, see James A. Weisheipl, *Ockham and Some Mertonians*, in: Mediaeval Studies, 30 (1968), pp. 163-213, at p. 190. The text is published in Marie Louise Roure, *La problématique des propositions insolubles au XIII^e siècle et au début du XIV^e, suivie de l'édition des traités de W. Shyreswood, W. Burleigh et Th. Bradwardine*, in: Archives d'histoire doctrinale et littéraire du moyen âge, 37 (1970), pp. 205-326 at pp. 285-326. This edition must be used with caution. On Bradwardine's theory, see Paul Vincent Spade, Insolubilia *and Bradwardine's Theory of Signification*, in: Medioevo, 7 (1981), pp. 115-134. The other major theories during this second stage of the literature were Roger Swyneshed's and William Heytesbury's. On Swyneshed's view, see Paul Vincent Spade, *Roger Swyneshed's* Insolubilia: *Edition and Comments*, in: Archives d'histoire doctrinale et littéraire du moyen âge, 46 (1979), pp. 177-220; and Paul Vicent Spade, *Roger Swyneshed's Theory of Insolubilia: A Study of Some of His Preliminary Semantic Notions*, in: Achim Eschbach and Jürgen Trabant, eds., *History of Semiotics*, (Foundations of Semiotics, vol. 7, Amsterdam 1983), pp. 105-113. On Heytesbury's theory, see Heytesbury, *op. cit.*

[19] See n. 7 above.

[20] Aristotle, *Sophistic Refutations*, 24, 180a38-b1: οὔτ' εἰ εὐορκεῖ τόδε ἢ τῇδε, ἀνάγκη καὶ εὐορκεῖν, ὁ δ' ὀμόσας ἐπιορκήσειν εὐορκεῖ ἐπιορκῶν τοῦτο μόνον, εὐορκεῖ δὲ οὔ. ("Neither, if he keeps *this* oath [to commit perjury], or keeps his oath in *this* way, is it necessary that he also keep his oath [without qualification]. Rather he who swears he will commit perjury keeps his oath in committing *this* perjury [with respect to a second oath] only; he does not keep his oath [without qualification].") The text is very obscure; in particular, Aristotle does not unambiguously say that the case he has in mind involves *two* oaths, the one oath to commit perjury and a second (perjured) oath fulfilling the first. I have reconstructed the case as it was generally interpreted by twelfth and thirteenth century Latin commentators. See Spade, *The Origins of the Mediaeval* Insolubila-*Literature*, pp. 302-306.

to say, "The argument is similar too concerning the same man's lying and speaking the truth at the same time."[21] Mediaeval authors saw here a reference to the Liar paradox. Hence it is not surprising to find that one of the earliest mediaeval approaches to *insolubilia* viewed them as fallacies *secundum quid et simpliciter.*

II

Here is a list of fifteen discussions of *insolubilia* from the period before Bradwardine. They are arranged in approximately chronological order, although it is impossible to be very confident about this, since the dating of some of the works is very uncertain. I have excluded from the list all discussions from this early period that are based on the passage in Aristotle's *Sophistici elenchi* but nevertheless fail to formulate a fully antinomial paradox like the Liar.[22]

(1) Anonymous, *Insolubilia* (late twelfth or early thirteenth century), Munich, Bayerische Staatsbibliothek, MS CLM 14458, fols. 39rb-40ra.[23]
(2) Robert Grosseteste, *Commentarium in Sophisticos elenchos Aristotelis* (1215-1229), Oxford, Merton College, MS 280, fols. 1ra-38va.[24]
(3) Anonymous, *Insolubilia* (c. 1225), Paris, Bibliothèque nationale, MS lat. 11412, fols. 88ra-91va.[25]
(4) Anonymous, *Insolubilia* (mid-thirteenth century), Paris, Bibliothèque nationale, MS lat. 16617, fols. 50v-54v.[26]
(5) Anonymous, *Insolubilia* (mid-thirteenth century), Paris, Bibliothèque nationale, MS lat. 16617, fols. 46v-50v.[27] The text has been attributed to William of Sherwood, but the attribution is not well founded.[28]

[21] Aristotle, *Sophistic Refutations*, 24, 180b2-3: ὅμοιος δ' ὁ λόγος καὶ περὶ τοῦ ψεύδεσθαι τὸν αὐτὸν ἅμα καὶ ἀληθεύειν. There are other paradoxes discussed in this passage too. But none of them is as developed as the example of the perjurer.
[22] There are many such discussions, down to the end of the thirteenth century and even beyond. See Spade, *The Origins of the Mediaeval* Insolubilia-*Literature*, pp. 302-307.
[23] Edited in De Rijk, *Some Notes on the Mediaeval Tract* De insolubilibus. See Spade, *The Mediaeval Liar*, item ix, p. 32.
[24] See S. Harrison Thomson, *The Writings of Robert Grosseteste, Bishop of Lincoln 1235-1253*, (Cambridge 1940), pp. 81-82; Spade, *The Mediaeval Liar*, item lix pp. 97-98. Unedited.
[25] Unedited, but discussed in De Rijk, *Some Notes on the Mediaeval Tract* De insolubilibus, pp. 93-98. See also Spade, *The Mediaeval Liar*, item xxi, pp. 43-44.
[26] Edited in H. A. G. Braakhuis, *The Second Tract on* Insolubilia *Found in Paris, B. N. Lat. 16.617: An Edition of the Text with an Analysis of Its Contents*, in: Vivarium, 5 (1967), pp. 111-145. See Spade, *The Mediaeval Liar*, item xx, pp. 42-43.
[27] Edited in Roure, *op. cit.*, pp. 248-261. See Spade, *The Mediaeval Liar*, item vi, pp. 26-28.
[28] See the discussion in De Rijk, *Some Notes on the Mediaeval Tract* De insolubilibus, p. 93.

(6) Lambert of Auxerre, *Logica* (1253-1257), tract vii ("De fallaciis"), in the discussion of the fallacy *secundum quid et simplicter.*[29]
(7) Richard of Sherwood, *Insolubilia* (thirteenth century), Cambridge, St. John's College, MS 100, fols. 135ra-138va.[30]
(8) Anonymous, *Quaestiones super Sophisticos elenchos* (1270-1280), qq. 94-95.[31]
(9) Simon of Faversham, *Quaestiones novae super libro Elenchorum* (c. 1280), q. 25.[32]
(10) John Duns Scotus, *Quaestiones super libros Elenchorum* (late thirteenth century), qq. 52-53.[33]
(11) Anonymous (Magister insolubilium), otherwise unknown, referred to in Thomas Chirmister's *Quaestiones in opus Prisciani*, q. 7, Cambridge, Gonville & Caius College, MS 688/645, fol. 21r. The manuscript is to be dated probably before 1307.[34]
(12) Anonymous, *Notandum* (before 1307), Cambridge, Gonville & Caius College, MS 688/645, fol. 76r.[35]
(13) Anonymous, *Tractatus de sophismatibus* (probably early fourteenth century, but possibly after Bradwardine), Bruges, Bibliothèque publique de la ville, MS 500, fols. 173ra-180va.[36]
(14) Walter Burley, *Insolubilia* (1302).[37]
(15) Richard of Campsall, *Quaestiones super librum Priorum analyticorum* (before 1307), q. 10.[38]

There is nothing in any way definitive about this list; it represents only those items that have come to my attention and that I am in a position to say something about. The list surely does not exhaust the *insolubilia*-literature before Bradwardine.[39] But it does allow one to get an idea of the diversity of contexts in which one finds this topic being discussed:

[29] See Lambert of Auxerre, *Logica* (*Summa Lamberti*), Franco Alessio, ed., (Firenze 1971), pp. 186-187. See Spade, *The Mediaeval Liar*, item xlv, pp. 77-78.
[30] Unedited. See *ibid.*, item lvii, pp. 94-95. Nothing is known of Richard of Sherwood. He is not to be confused with William of Sherwood.
[31] See *Incertorum auctorum quaestiones super Sophisticos elenchos*, Sten Ebbesen, ed., (Corpus philosophorum Danicorum medii aevi, vol. 7, Copenhagen 1977), pp. 221-226. This text is not discussed in Spade, *The Mediaeval Liar.*
[32] See Simon of Faversham, *Questiones super libro elenchorum*, Sten Ebbesen, *et al.*, eds., (Studies and Texts, vol. 60, Toronto 1984), pp. 165-167. This text is not discussed in Spade, *The Mediaeval Liar.*
[33] In John Duns Scotus, *Opera omnia*, Luke Wadding, ed., (Lyon: Laurentius Durand, 1639; reprint Hildesheim 1968), vol. 1, pp. 268-269. See Spade, *The Mediaeval Liar*, item xxxvii, pp. 65-66.
[34] See *Ibid.*, item i, p. 19.
[35] This very brief note is edited *ibid.*, item xiv, pp. 37-38.
[36] See *ibid.*, item xix, pp. 41-42. Unedited.
[37] Edited in Roure, *op. cit.*, pp. 262-284. See Spade, *The Mediaeval Liar*, item lxvii, pp. 111-113.
[38] See *The Works of Richard of Campsall*, Edward A. Synan, ed., vol. 1: *Quaestiones super librum Priorum analyticorum*, (Studies and Texts, vol. 17.1, Toronto 1968), pp. 162-177. See Spade, *The Mediaeval Liar*, item liv, p. 91.
[39] See section VIII of this paper, below.

(a) In literal commentaries on the *Sophistici Elenchi*—for example, Grosseteste's (item 2 above). This is not surprising in view of the role the *Elenchi* played in the *insolubilia*-literature. It is curious, however, that Grosseteste has little to say about Aristotle's man who lies and speaks the truth at the same time. Instead of trying to force that example into the pattern of Aristotle's "perjurer"—example, as many authors tried rather unsuccessfully to do,[40] Grosseteste does the opposite, casting the "perjurer"—example in the mold characteristic of the Liar. Whereas Aristotle seems to have the perjurer take two oaths, one of them ("I swear I will commit perjury") in the future tense, Grosseteste has him take only one oath, and that in the present tense: "By God, I am committing perjury". And that is the only oath he takes.

(b) In collections of questions on the *Elenchi.* For instance, Scotus' (item 10 above), Simon of Faversham's (item 9), and the anonymous collection listed as item 8.

(c) In a logical *Summa* (Lambert of Auxerre's, item 6), in the section discussing the fallacy *secundum quid et simpliciter.* Curiously, there is no analogous discussion in those other two great thirteenth century *Summulae* of logic, William of Sherwood's[41] and Peter of Spain's.[42]

(d) In a *Sophismata* not directly related to Aristotle's *Sophistici elenchi* (item 13 above).

(e) In a little note jotted in a manuscript (item 12).

(f) In several independent treatises devoted exclusively to the topic (items 1, 3-5, 7, 14 above).

(g) And in various other contexts as well (items 11, 15).

III

In all this variety of material, we can distinguish five main approaches during this pre-Bradwardinian period. Sometimes they were

[40] See Spade, *The Origins of the Mediaeval* Insolubilia-*Literature*, pp. 302-306.

[41] See Martin Grabmann, *Die Introductiones in logicam des Wilhelm von Shyreswood*, (Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Philos.-hist. Abt., 1937, Heft 10, Munchen 1937), translated in Norman Kretzmann, *William of Sherwood's Introduction to Logic*, (Minneapolis 1966). Sherwood does, however, mention *insolubilia* in his *Syncategoremata.* See J. Reginald O'Donnell, ed, *The Syncategoremata of William of Sherwood*, in: Mediaeval Studies, 3 (1941), pp. 49-93 at p. 58; translated in Norman Kretzmann, *William of Sherwood's Treatise on Syncategorematic Words*, (Minneapolis 1968), p. 55.

[42] Peter of Spain, *Tractatus, Called Afterwards Summule logicales*, L. M. De Rijk, ed., (Assen 1972).

combined in more or less ingenious ways, and sometimes it would be perhaps appropriate to subdivide them. But basically, there are five main views.

First, as might be expected, there is the "Aristotelian" solution that views the paradoxes as instances of the fallacy *secundum quid et simpliciter*. Significantly, this approach is *not* taken by item 1 in the above list, the earliest known mediaeval treatment that actually proposes a solution,[43] although the author of that work does know about the fallacy and appeals to it in his discussion.[44] Neither is it the approach taken by items 4 or 14 in the list. On the other hand, it is adopted by items 2-3, 5-10, perhaps 11,[45] by 12-13 and by 15. Moreover, the fallacy *secundum quid et simpliciter* continued to be appealed to in the context of *insolubilia*—although not with such regularity— even well after the early or "seminal" period that is the topic of this paper.[46]

On the other hand, many of our authors use this fallacy in ways quite unlike what Aristotle had in mind. Aristotle at least suggested that the Liar sophism is to be regarded as false *simpliciter* but true *secundum quid*.[47] On the other hand, some authors, such as the author of the little note that is item 12 on the above list,[48] appear to have held that the Liar sentence was neither true nor false *simpliciter*, but rather both true and false *secundum quid*. This seems different from what Aristotle had mind.[49]

[43] This suggests that the historical link between the *Elenchi* and the *insolubilia*-literature is more complicated than it first appears. See Spade, *The Origins of the Mediaeval* Insolublia-*Literature*, p. 307.

[44] See section VII of this paper, below.

[45] The brief reference to the "Magister insolubilium" suggests this approach, but is too meager to allow any definitive judgment about his view.

[46] For example, by Bradwardine (see Roure, *op. cit.*, pp. 305-307, para. 7.11-7.14), and by Peter of Ailly (see Spade, *Peter of Ailly*, pp. 93-94, para. 378-383). In these authors, however, the role of the fallacy may be described as merely "honorary". The authority of Aristotle is preserved, but the real emphasis of the theory is elsewhere.

[47] Aristotle, *Sophistical Refutations*, 25, 180b5-7: κωλύει δ' αὐτὸν οὐδὲν ἁπλῶς μὲν εἶναι ψευδῆ, πῇ δ' ἀληθῆ ἢ τινός, καὶ εἶναι ἀληθῆ τινά, ἀληθῆ δέ μή. ("Nothing prevents its being false absolutely but true *somehow*, or [true] *of* something, or being true *with respect to* something but not true [without qualification].")

[48] See Spade, *The Mediaeval Liar*, item xiv, p. 38: "Et dicendum quando quaeritur 'aut dicit verum aut non verum,' dicendum quod neque sic neque sic, sed dicit hoc non verum." ("And it is to be said that when one asks 'Does he speak truth or untruth?', it is to be said that it is neither the one nor the other. Rather he speaks *this* untruth.")

[49] Shortly after Bradwardine, Richard Kilvington held a view like this. See his *Sophismata*, soph. 48. Norman Kretzmann and Barbara Ensign Kretzmann are prepar-

Again, some authors—for instance, Lambert of Auxerre (item 6 on the list) and the author of the text ascribed, without convincing evidence, to William of Sherwood (item 5)—appealed to the Aristotelian fallacy by saying that the Liar sentence can contain no term that refers *simpliciter* to, or supposits *simpliciter* for, the whole sentence, but only *secundum quid.* Here it is not truth or falsehood *secundum quid* but rather supposition or reference *secundum quid* that is involved. This is clearly an attempt to combine the Aristotelian view, that the way to solve these paradoxes is by pointing out a fallecy *secundum quid et simpliciter*, with some kind of restriction on or rejection of self-reference.[50]

The fallacy *secundum quid et simpliciter* thus became a kind of authoritative framework that was imposed somewhat artificially on quite diverse attempts to solve the paradox. To the end of the Middle Ages, authors proved to be very adept at showing that the most marvelous theories were really what Aristotle had meant all along in that passage in the *Elenchi.*

IV

A second main approach to *insolubilia* during this early period is the view Walter Burley (in item 14 in the above list) calls "*transcasus*". The term is obscure in this context. It appears to be a literal translation of the Greek μεταπτῶσις ("change").[51] In Stoic logic, propositions that could change their truth value over time were called μεταπίπτοντα.[52] And indeed, Burley uses the term '*transcasus*' in exactly this way in his *Obligationes*,[53] and defines it this way in his *Insolubilia.*[54]

ing an edition and translation of Kilvington's *Sophismata*, together with a commentary. Simon of Faversham explicitly argues against such a theory (*op. cit.* p. 166.55-59), on the grounds that a sentence true in one respect but false in another is a "multiplex" (that is, ambiguous) sentence, whereas the Liar sentence is not like that. The Liar paradox cannot be solved simply by saying it involves an equivocation.

[50] Simon of Faversham, *op. cit.*, p. 167.88-95, and John Duns Scotus, *op. cit.*, q. 52, p. 269, perhaps have a similar view, although it is not expressed explicitly in terms of the notions "*simpliciter*" and "*secundum quid*".

[51] Sten Ebbesen first called my attention to this fact.

[52] See Michael Frede, *Die stoische Logik*, (Abhandlungen der Akademie der Wissenschaften in Göttingen, Philologisch-Historische Klasse, dritte Folge, Nr. 88; Göttingen 1974), pp. 44-48.

[53] See Romuald Green, *The Logical Treatise "De obligationibus": An Introduction with Critical Texts of William of Sherwood and Walter Burley*, (St. Bonaventure, NY forthcoming), para. 3.80: "Ad tertium dicitur, negando istam 'A est' et quando negatur, tunc positum est verum. Sed quando proponebatur, fuit falsum, et ideo accidit

Nevertheless, the actual theory he presents in the *Insolubilia* under this name appears, at least at first, to be somewhat different. Although there is still an appeal to different times, the notion of a *change* of truth value over time seems to play no role. The theory goes like this:

If I utter the sentence 'I am saying a falsehood', the verb in that sentence, even though it is grammatically in the present tense, nevertheless refers to a time before the utterance of that sentence, so that despite the grammar, the sentence really means something like 'I was saying a falsehood a moment ago'. My sentence is thus either straightforwardly true or straightforwardly false, depending on whether I did or did not utter a falsehood at the earlier time referred to.[55]

Burley thinks this view is important enough to reject it explicitly,[56] and so does Bradwardine.[57] Nevertheless we know of only two texts that actually maintain this theory during the period we are consider-

transcasus." ("I reply to the third by denying 'It is *A*'. And when it is denied, then the positum is true. But when it was proposed it was false, and therefore a *transcasus* occurs.")

[54] Roure, *op. cit.*, p. 270, para. 2.07: "Et est transcasus quando aliqua propositio mutatur a veritate in falsitatem vel e converso." ("And there is a *transcasus* when some sentence is changed from truth to falsehood or conversely.")

[55] *Ibid.*: "Unde dicunt quod me dicente me dicere falsum, si postea quaeratur: aut dico verum aut falsum, dicunt quod falsum, quia aliud est tempus in quo dico aliquid et pro quo dico aliquid. Ideo si dico me dicere falsum, dico hoc pro tempore precedente; et quia in tempore precedente nihil dixi, ideo in dicendo me dicere falsum, dico falsum, quia dico aliter quam est in re." ("Hence they say that when I say I am saying a falsehood, if afterwards it is asked whether I say a truth or a falsehood, they say that [I say] a falsehood. For the time *in* which I say something is other than the time *about* which I say something. Therefore, if I say I am saying a falsehood, I say it about a preceding time. And because in the preceding time I said nothing, therefore in saying I am saying a falsehood I say a falsehood, because I say otherwise than is really the case.")

[56] *Ibid.*, p. 271, para. 2.08. Nevertheless, Burley thinks the theory is more plausible than others. See *ibid.*, para. 2.07: "et ista opinio est probabilior inter alias." On the reason for this, see n. 76 below.

[57] *Ibid.*, p. 294, para. 5.04: "Quarta oppinio est restringentium tempus, ut premissum est. Qui sic nominantur pro tanto quia dicebant quod Sorte dicente: *Sortes dicit falsum*, hoc verbum *dicit* licet sit presentis temporis, tamen debet intelligi pro tempore sive instanti immediate precedente tempus prolationis." ("The fourth opinion is that of those who restrict the time, as was stated before. They are so called insofar as they used to say that when Socrates says 'Socrates is saying a falsehood', the verb 'is saying', although it is in the present tense, nevertheless should be understood for the time or instant immediately preceding the time of utterance.") Note that Bradwardine does not use the term '*transcasus*' for this view, but rather describes it as the the theory of the "*restringentes tempus*" ("those who restrict the time"). Bradwardine discusses this theory in the imperfect tense (*dicebant*, they used to say), suggesting perhaps that by his time the view was no longer current.

ing.[58] One is Simon of Faversham's discussion (item 9 above), although Simon devotes only a few lines to affirming this view. He provides no elaboration and absolutely no argument in favor of this approach. The other text offers a fuller picture. It is item 4 in the above list, from the mid—thirteenth century. The anonymous author of that text argues for the theory of *transcasus* on the grounds that "discourse can take place about anything that is prior according to nature to the discourse that takes place about it, and about only such a thing,"[59] and "everything that is prior in time is prior by nature."[60]

Bradwardine offers a different argument for this theory, although his wording ("can give") suggests that perhaps this argument was not actually used by the theory's proponents:[61]

> And they can give the following reason for this. For language exists in man in order to express what the mind conceives. Therefore, man expresses *for* the time or instant in which he conceives. But he naturally conceives before he speaks. Therefore, he expresses for the time or instant before the time of utterance.

Neither of these reasons is especially persuasive. Perhaps some additional motivation for the theory may have resulted from tampering with tenses in an attempt to make the Liar fit what Aristotle had said about his "perjurer"—example.[62] But I am not at all confident of this. First, Aristotle had the perjurer speak in the future tense, and I do not see how we can get from that to the past time involved in the theory of *transcasus*. Furthermore, the anonymous item 4 in our list, in which *transcasus* is maintained, is one of the few early texts that does *not* appeal to the fallacy *secundum quid et simpliciter*, in the discussion of which Aristotle introduces his perjurer who speaks in the future tense.

A more likely rationale for the theory might have arisen from viewing insolubles as emerging in the context of a *disputatio de*

[58] It is possible, and perhaps even plausible, to read parts of Scotus' discussion this way as well. But it is not unequivocally required by his actual text. See section V below.

[59] Braakhuis, *op. cit.*, p. 137.29-30: "sermo potest fieri de quolibet quod est prius secundum naturam ipso sermone facto de ipso, et de solo tali."

[60] *Ibid.*, p. 138.5-6: "omne quod est prius tempore est prius natura." By themselves, these considerations do not imply the view Burley describes. Nevertheless, rightly or wrongly, the anonymous author does seem to hold that view for these reasons.

[61] Roure, *op. cit.*, p. 294, para. 5.04: "et possunt ad hoc habere talem rationem, quia loquella est in homine ad exprimendum mentis conceptum, ergo homo exprimit pro tempore sive instanti in quo concipit, sed naturaliter prius concipit quam loquitur. Ergo exprimit pro tempore sive instanti priori tempore prolationis."

[62] See Spade, *The Origins of the Mediaeval* Insolubilia-*Literature,* pp. 303-306 and p. 307 n. 64.

obligationibus.[63] There, "the time *for* which something is conceded is earlier than the time *in* which something is conceded."[64]

V

A third approach to the Liar is taken by John Duns Scotus (item 10 on our list). Scotus adopts the Aristotelian framework of the fallacy *secundum quid et simpliciter*, but combines this with a distinction between what he calls the "exercised act" (*actus exercitus*) and the "signified act" (*actus significatus*). In the standard case of the sentence 'Socrates is saying a falsehood', the signified act is the act Socrates is talking about, that is, what Socrates *says* he is doing—namely, saying a falsehood. The exercised act is what Socrates actually does in his utterance. Scotus thinks that what Socrates actually does in this case is to say a truth. Therefore, the act he is talking about, the *actus significatus*, must be an act of speaking falsely. As he puts it, "there is falsehood in the signified act and truth in the exercised act."[65]

In order to avoid attributing both truth and falsehood to the same act of speaking, this theory must hold that the exercised act and the signified act are *two* acts, not one. And indeed, Scotus does hold this, as we shall see.

The application of the notions *actus exercitus/actus significatus* to the context of the paradoxes seems to have been an approach that had some circulation. Bradwardine mentions a view perhaps like Scotus', but rejects it:[66]

[63] On such *disputationes de obligationibus*, see Eleonore Stump, *Obligations: From the beginning to the early fourteenth century*, in: *The Cambridge History of Later Medieval Philosophy*, Ch. 16, Part A, pp. 315-334; and Paul Vincent Spade, *Obligations: Developments in the fourteenth century*, in: *The Cambridge History of Later Medieval Philosophy*, Ch. 16, Part B, pp. 335-341.

[64] See Walter Burley, *Tractatus de obligationibus*, in Green, *op. cit.*, para. 3.153: "...sed tempus pro quo conceditur aliquid, est prius quam in quo conceditur aliquid." See also n. 55 above, where there is a claim of distinction, although no claim about priority.

[65] Scotus, *op. cit.*, q. 53, ad 1, p. 270: "...sed in proposito falsitas est in actu sign⟨ific⟩ato , et veritas in actu exercito." The distinction between exercised and signified acts is reminiscent of the modern distinction between use and mention. But they are not the same distinction.

[66] See Roure, *op. cit.*, p. 296, para. 5.08: "Octava oppinio est *distinguentium*. Qui ob hoc tale nomen acceperunt quia Sorte dicente *Sortem dicere falsum* distinguebant de *dicere* penes equivocationem. Potest enim significare dicere exercitum vel conceptum. Et vocant 'dicere exercitum' dicere quod est in exercitio, et est illius quod est in dici (in dici] indici *ed.*) et non est dictum complete; 'dicere vero conceptum', dicunt, cum homo prius dixerit aliquid vel aliquale et instanti post dicat se dicere illud vel tale...."

> The eighth opinion is that of the "distinguishers", who take such a name because of the fact that, when Socrates says that Socrates is saying a falsehood, they distinguish the equivocation in 'say'. For it can signify an *exercised* saying or a *conceived* saying. The saying that is in the process of being said and is not completely said they call the *exercised* saying. But when a man earlier says something, or in some way, and at a later instant says that he says that, or in that way, they call it the *conceived* saying.

Note that Bradwardine contrasts the exercised act with the *conceived* act; he does not use the term '*actus significatus*' as Scotus did.[67] Note also that Bradwardine interprets the theory he describes in such a way that it is committed to a form of *transcasus*. Scotus' theory is perhaps not committed to this. He does say:[68]

> Now that in this case there is falsehood absolutely, this is clear from the fact that an expression is said to be absolutely false because it is a *sign* of the false. For one who begins to speak like this [namely, 'I am saying a falsehood'] says nothing that might be false. If he were to say that a man is an ass, or something like that, [then] this ['I am saying a falsehood'] would be true.

But, although it is a plausible interpretation, there is nothing explicitly in this passage (or anywhere else in Scotus' discussion) to indicate that the "other" falsehood that would make 'I am saying a falsehood' true would have to be a *prior* falsehood; it might very well, for instance, be a falsehood the speaker is just about to say.[69]

Later in the fourteenth century, the distinction between exercised and signified acts is mentioned by Marsilius of Inghen—and through him by Peter of Ailly—in the context of the paradoxes, although it is not clear that they had Scotus' doctrine particularly in mind.[70] Indeed Scotus himself makes it clear that he is not the first to apply the distinction between exercised and signified acts to the paradoxes:[71]

[67] Given the close linkage between the notions of signification and thought in mediaeval semantics, this terminological disparity is perhaps not especialy significant. See Paul Vincent Spade, *The Semantics of Terms*, in: *The Cambridge History of Later Medieval Philosophy*, Ch. 9, pp. 188-196.

[68] Scotus, *op. cit.*, q. 53, p. 269: "Quod autem circa ipsum sit falsitas simplicter, hoc patet per hoc, quod oratio dicitur simpliciter falsa, quia est signum falsi, quoniam sic incipiens loqui, nihil dicit, quod sit falsum. Si enim diceret hominem esse asinum, vel aliquid huiusmodi, haec esset vera."

[69] In his only argument *ad oppositum* in q. 53, Scotus says "...sed haec oratio est falsa 'Ego dico falsum' quam dicit incipiens sic loqui, cum nihil prius dixit, et cetera." (... but the expression 'I am saying a falsehood', which he says who begins to talk this way, is false *since he said nothing earlier*, and so on.") Scotus agrees with this side of the question, but he does not explicitly say anything one way or the other about this argument for it.

[70] See Spade, *Peter of Ailly*, p. 69, par. 262, and p. 143 nn. 692-693.

[71] *Ibid.*: "Quod autem sic incipiens loqui sit verus secundum quid diversimode ponitur a diversis. Quidam enim ponunt quod est verus secundum quid quia circa ali-

> But that he who begins to speak like this is true[72] in a certain respect is maintained in different ways by different people. For some maintain that he is true in a certain respect because an act of speaking is exercised truly about some false expression. In another way, it is said that he who says this is true in a certain respect because in the exercised act the case is understood through the proposition to be as is really the case. But because it is not from the exercised act that an expression is said to be absolutely true or false, therefore such an expression is said to be true *in a certain respect*. For in the exercised act it is understood that he exercises an act of "saying" about a falsehood, and that is true in reality.

Scotus' theory is an amalgam of several lines of thought. There is, first of all, the distinction between exercised and signified acts, which is what I am emphasizing here. In addition, there may or may not also be a version of *transcasus*. But in any case, insofar as the theory is committed to saying that the exercised act and the signified act are two, not one, Scotus is also committed to some kind of restriction on the possibility of self-reference. This brings us to the next main theory I want to discuss.

VI

There remain two other important views during this early period of the *insolubilia*-literature. The first is called "*restrictio*" ("restriction"), the view of the "*restringentes*". This view rejects the possibility of self-reference in some or all cases. The kind of "reference" at stake here is "supposition"—a property of categorematic terms that occur in a sentence.[73] Thus the characteristic question with which this view was concerned, and which it answered in the negative, was whether "a part can supposit for the whole of which it is a part".

Some authors also considered more complicated kinds of pathological reference, and denied not only that a term in a sentence P can supposit for P itself, but also that it can supposit for the contradictory of P, or for another sentence Q that is logically equivalent to P, or

quam orationem falsam exercetur vere actus dicendi. Dicitur aliter quod sic dicens est verus secundum quid, quia actu exercito per propositionem intelligitur sicut est in re. Sed quia ab actu exercito non dicitur oratio simpliciter vera vel falsa, ideo talis oratio dicitur secundum quid vera. Actu enim exercito intelligitur, quod ipse exercet actum dicendi circa falsum, et illud in re est verum."

[72] Scotus speaks, both here and throughout the question, of the *speaker*, and not just the speaker's proposition, as being true or false. Indeed the title of his q. 53 is: "Utrum incipiens sic loqui 'Ego dico falsum' sit simpliciter verus in dicendo?" ("Whether he who begins to speak like this 'I am saying a falsehood' is absolutely true in [so] saying?") The justification for this usage is given *ibid.*: "Ad oppositum. Dicens denominatur a qualitate sui dicti...." ("On the opposite side: A speaker is denominated by the truth value of what is said....")

[73] On supposition, see Spade, *The Semantics of Terms*.

that in turn contains a term that supposits for *P*, and so on.[74] They were worried not just about the case in which Socrates says only 'Socrates is saying a falsehood', but also about cases where, for instance:

(a) Socrates says only 'What Plato is saying is true', and Plato says only 'What Plato is saying is false'. Here Plato's sentence is an immediate and direct insoluble. But no matter how one chooses to handle Plato's sentence, there is still a problem with Socrates' sentence. Socrates' sentence is simply the contradictory negation of Plato's sentence, so that the two must presumbaly have opposite truth values. Hence, by saying that its contradictory is true, Socrates' sentence seems in effect to be saying that it itself is false, and so in this indirect way to be an insoluble too.

(b) Socrates says only 'What Plato is saying is false' and Plato also says 'What Plato is saying is false'. Again, Plato's sentence is a direct insoluble. But Socrates' sentence is not; it does not refer to *itself*; it refers to Plato's sentence, a different "token" of the same "type".[75] But however one handles Plato's sentence, the two sentences are duplicates of one another, both syntactically and semantically (they refer to exactly the same thing and make exactly the same claim about it, that it is false), so that they are presumably logically equivalent and have the same truth value. Hence, by apparently saying that its logical equivalent is false, Socrates' sentence seems in effect to be saying that it itself is false too.

(c) Socrates says 'What Plato is saying is false' and Plato says 'What Socrates is saying is true'. Here, apparently, what Socrates says is true if and only if what Plato says is false—that is, if and only if what Socrates says is *not* true. Hence, what Socrates says is true if and only if it is not true, which is paradoxical.

In such cases the *restringentes* said that this kind of reference or supposition simply cannot occur. When Plato, for instance, says only 'What Plato says is false', the term 'false' there cannot refer to or sup-

[74] See Spade, *The Mediaeval Liar*, item xxx, pp. 55-56 (Henry of England), and item lxii, pp. 101-102 (Roger Roseth). Virtually nothing is known about the mysterious "Henry of England"; even the name is uncertain. Roger Roseth wrote probably slightly after Bradwardine. I have been unable to find any author who wrote *definitely* before Bradwardine and who rejected these more complicated forms of referential pathology. There must have been some, however, since Bradwardine takes great pains to argue against such a view. See Roure, *op. cit.*, pp. 287-293, para. 2.00-4.11.

[75] Many mediaeval authors held that it is the sentence-token, not the type and not some "proposition" expressed by the sentence, that is the bearer of truth value.

posit for Plato's sentence itself, but must instead refer only to *other* falsehoods. Hence Plato's sentence in effect means that Plato says something false other than that very sentence. But by hypothesis Plato doesn't say *anything*—true or false—other than that very sentence. Hence, the sentence he does say is just straightforwardly false. Similar treatments can be applied to (a)-(c) above.

It is easy to see how this approach might readily be combined with the theory of *transcasus*, as in fact it was combined in the anonymous item 4 on our list, and perhaps in Scotus' theory as well. Indeed, *transcasus* entails *restrictio*. *Restrictio* says that the spaeker is talking about some other sentence than the one he is in the process of uttering. *Transcasus* just narrows down the possibilities: he is talking about what he said just a moment ago.[76]

Restrictio was a popular approach in the mediaeval *insolubilia*-literature, even well after the end of the early period that is the topic of this paper. Whereas *transcasus*, for instance, died out fairly early, *restrictio* was one of the enduring views.[77]

It is worth pointing out the obvious similarity between the theory of *restrictio* and modern theories that avoid the paradoxes by denying self-reference—often with an appeal to a hierarchy of types or levels of language. And the same criticism that is often levelled against these modern theories can be levelled also against at least a broad application of the theory of *restrictio*: all these theories are too sweeping. They rule out not only the vicious cases of self-reference that we want to do something about, but also cases of quite innocuous self-reference that we have no antecedent reason to want to rule out. For instance, I can quite intelligibly and even truthfully say 'What I am now saying is in English', or 'This sentence has five words'. Yet these theories would rule out such sentences.

Some mediaeval authors were aware of this criticism and raised it.[78]

[76] This seems to be the reason Burley says that *transcasus* is a plausible view, even though in the end he rejects it. (See n. 56 above.) For Burley's own theory is a form of *restrictio*. See below.

[77] See Spade, *The Mediaeval Liar*, index under '*restringentes*'.

[78] See, for instance, Burley in Roure, *op. cit.*, p. 270, para. 2.05: "... si Sortes incipiat sic loqui: *aliquid dicitur a Sorte*, tunc est hec propositio vera: *aliquid dicitur a Sorte*; ergo est vera pro aliquo supposito subjecti, sed pro nullo supposito subjecti potest esse vera nisi pro hac oratione *aliquid dicitur a Sorte*, quia Sortes nihil aliud dicit. Ergo subjectum huius: *aliquid dicitur a Sorte* potest supponere pro hac tota." ("... If Socrates begins to speak as follows 'Something is said by Socrates', then this proposition 'Something is said by Socrates' is true. Therefore, it is true for something for which the subject supposits. But it can be true for nothing for which the subject supposits unless it is for this

It is perhaps because of this that we find the theory of *restrictio* in two forms in the *insolubilia*-literature, a strong form and a weak form. Strong *restrictio* rules out all self-reference whatever,[79] and leaves itself open to the criticism just mentioned. Weak *restrictio* rules out self-reference only in the case of *insolubilia*. It is sometimes difficult or even impossible to tell whether an author is maintaining *restrictio* in its strong or its weak form. Nevertheless, at least this much can be said. Before Bradwardine, the strong form of *restrictio* was maintained by item 4 in the above list, the anonymous mid-thirteenth century text that adopts the theory of *transcasus*; by item 5, the anonymous text sometimes ascribed to William of Sherwood; by item 9, Simon of Faversham, with a qualification;[80] and by Scotus, item 10, with the same qualification.[81] Curiously, although the author of item 5 accepts the *restringentes*' rejection of self-reference, he does not think this alone will adequately solve the paradoxes.[82] His own theory in effect accepts their view, but is cast in terms of supposition *simpliciter* and supposition *secundum quid*. Strong *restrictio* is also accepted by item 13 on the above

expression 'Something is said by Socrates', because Socrates says nothing else. Therefore, the subject of 'Something is said by Socrates' can supposit for this whole [sentence].'')

[79] And perhaps also the other kinds of reference discussed in (a)-(c) above.

[80] Simon of Faversham, *op. cit.*, p. 167.88-95. Simon, does not rule out such supposition or reference altogether, but says that it is ''improper''. A term contained in a sentence cannot properly, *as a part*, supposit for that whole sentence. Yet, *as a universal whole*—that is, as a general term referring to a whole range of things—it may indeed supposit for that whole sentence if that sentence is included in the range. This is a remarkably modest restriction on self-reference. Nevertheless since, whatever it comes to, it is intended to apply to *all* sentences and not just to *insolubilia*, I have included Simon among those who hold a version of strong *restrictio*.

[81] Scotus, *op. cit.*, q. 52, p. 269. For the qualification, see the preceding note.

[82] See Roure, *op. cit.*, p. 253, para. 4.10: ''Licet autem restringentes bene dicant quo (quo] quoque *ed.*) ad hoc quod pars non potest supponere pro toto, male tamen solvunt, quia, cum concedunt totum processum usque ad ultimum argumentum, concedunt opposita, quia respondendo concedunt me dicere falsum, et cum proceditur sic 'ergo falsum est me dicere falsum, ergo non dico falsum', male negant illud idem. Item, hec solutio non est secundum Aristotelem; propterea concedendum est quod restrictio sit via in solutionem veram, ut patebit, et non vera solutio.'' (''But although the *restringentes* are well-spoken with respect to the fact that a part cannot supposit for the whole, nevertheless they give a bad solution. For, when they grant the whole [reasoning-] process up to the last argument, they grant opposites. For in their reply they *grant* that I say a falsehood [when I say only 'I am saying a falsehood']. And when [their] argument goes on as follows: 'therefore it is false that I say a falsehood; therefore, I do not say a falsehood', they wrongly *deny* the same thing. Again, this solution is not in accordance with Aristotle. Therefore, it is to be granted that *restrictio* is the *way* to the true solution, as will be clear, and not the true solution [itself].'')

list, the anonymous early fourteenth century *Tractatus de sophismatibus*.[83]

On the other hand, the following items on our list of early discussions maintain *at least* the weak version of *restrictio*: item 6, Lambert of Auxerre who, like the author of item 5, puts his theory in terms of supposition *simpliciter* and supposition *secundum quid*; item 12, the anonymous early fourteenth century note; item 14, Walter Burley from the early fourteenth century, who clearly does *not* hold the stronger form of *restrictio*;[84] and item 15, Richard of Campsall, who also clearly does not hold the stronger version.

The following authors on our list *rejected* one or the other form of *restrictio*: item 3, the anonymous text from c. 1225, which rejects even weak *restrictio*; item 8, the anonymous questions on the *Elenchi* from around 1270-1280, where in q. 95 the author rejects even weak *restrictio*; and item 14, Burley, who rejects only the strong form of *restrictio*.[85]

The strong form of the theory is indeed too strong; it is open to the objections mentioned above. On the other hand, the weak form of the theory in practice tended to be *ad hoc*. Self-reference is possible except in *insolubilia*. But in the absence of any precise and independent characterization of insolubility, this in practice meant that self-reference is possible except when it leads to contradiction. In short, self-reference is possible except when it is is not. That is certainly true, but it is not a theory—it is a tautology.[86]

[83] There are some palaeographical problems with reading this text in microfilm, so that I cannot be altogether sure of this claim. Nevertheless, as I read the text, it maintains strong *restrictio*. Henry of England, whose dates are unknown (see n. 74 above), also maintained strong *restrictio*, as did Richard Brinkley in the later fourteenth century (for astonishingly bad reasons). Brinkley's text is listed as anonymous in Spade, *The Mediaeval Liar*, item x, pp. 33-34, and is edited in Paul Vincent Spade, *An Anonymous Fourteenth-Century Treatise on 'Insolubles': Text and Study*, M. S. L. Dissertation, Toronto, Pontifical Institute of Mediaeval Studies, 1969. On Brinkley, see Gedeon Gál and Rega Wood, *Richard Brinkley and His* Summa logicae, in: Franciscan Studies, 40 (1980), pp. 59-101. It was Gál and Wood who first identified my "anonymous" treatise as part of Brinkley's *Summa logicae*.

[84] Burley's theory of *insolubilia* was taken over almost exactly by Ockham. See Paul Vincent Spade, *Ockham on Self-Reference*, in: Notre Dame Journal of Formal Logic, 15 (1974), pp. 298-300.

[85] On Simon of Faversham and Scotus, both of whom might justifiably be included in this list, see nn. 80-81 above.

[86] See Spade, *Ockham on Self-Reference*.

VII

The fifth and last of the major early theories of *insolubilia* is the theory of "*cassatio*", the view of the "*cassantes*", those who "nullify" or "cancel". On this theory, when one utters an insoluble sentence, he is not really saying anything. '*Nil dicis*' ("You are saying nothing") is the phrase characteristic of this view. This seems perhaps to mean that what is said "lacks sense" or "fails to make a statement".

This view is a popular modern theory although once again, in the absence of any exact and independent characterization of the paradoxes, the theory tends to be trivial: sentences make statements or have sense except when they cannot.

Cassatio was maintained by two of our early texts: item 1, the anonymous text from the late twelfth or early thirteenth century; and item 3, the anonymous text from c. 1225.[87] It is significant, I think, that two of the earliest known treatments of *insolubilia* take this quite non-Aristotelian approach to the paradoxes. Item 3, around 1225, says that this view—which the author does not call *cassatio* but rather "*interemptio actus*" ("abolishing the act") is "according to the common judgment" (*secundum commune judicium*).[88] Formerly I took this as evidence for the widespread acceptance of this view in the early thirteenth century. But now I think the author meant only that the view of the *cassantes* agrees with what the "common man" would say. The author makes a rather curious appeal to the "man-on-the-street": If you were to approach a "*rusticus*" and say 'What I am saying is false', he would reply that you are not saying anything: "*Nil dicis.*"[89]

Item 1 on our list, the earliest extant complete treatment of *insolubilia*, already recognizes that some distinctions and refinements are necessary in order to make the view plausible. The author says that in the case of an insoluble we have to deny that there is an act of saying anything, since in general, with respect to any judgment or proposition (*enuntiabile*), we have to deny everything that would make it insoluble.[90] He then raises an objection:[91]

[87] In Spade, *The Mediaeval Liar*, p. 43, I wrongly said that this text rejects *cassatio*.
[88] See *ibid.*
[89] *Ibid.*, p. 44.
[90] See De Rijk, *Some Notes on the Mediaeval Tract* De insolubilibus, pp. 105.32-106.2.
[91] *Ibid.*, p. 106.3-16: "Sed contra. *Dicere* nil aliud est quam prius asserere et postea proferre. Sed assertio non est cassanda, cum ex ipsa non sequatur inconveniens. Dato enim quod asseratur et non proferatur, inpune potest sustineri quod sit falsum. Similiter prolatio non est cassanda nec solet cassari. Et ita cum assertio non sit cassan-

But on the contrary: "To say" is nothing else but first to assert[92] and afterwards to utter. But the assertion is not to be cancelled, since an inconsistency does not follow from *it*. For given that ['I am saying a falsehood'] is asserted and not uttered, it can be maintained with impunity that it is false. Likewise, the utterance is not to be cancelled—nor is it usually cancelled. And so, since the assertion is not to be cancelled, or the utterance, and these two are [what it is] "to say", [therefore], the saying is not to be cancelled. And so, if something is to be cancelled, and the assertion is not to be cancelled since it is prior and an inconsistency does not follow from it, it remains, as it seems, that the utterance is to be cancelled. And that is not usually done.

Solution: Just as the definition 'rational animal' is denied of everything other than man, yet its parts are not denied,[93] likewise 'assertion with utterance' is denied of this proposition (*enuntiabile*) [and] yet its parts are not denied. Hence the assertion is not to be cancelled absolutely, but in a respect that it has to the utterance. Hence neither is to be cancelled absolutely, but the one with respect to the other.

Parts of this passage are obscure. But it is clear at any rate that the charge, which the anonymous item 5, attributed to William of Sherwood, levels against *cassatio*, that it simply goes contrary to sensation,[94] is altogether too facile. The above passage explicitly says that it is not the sensible utterance that is cancelled or abolished. Nevertheless, this unfair objection was picked up by Burley[95] and by Thomas Bradwardine, who adds the claim that the view also "contradicts the understanding,"[96] which likewise seems unfair. Through Bradwardine, whose survey of previous opinions was quoted more or less ver-

da nec prolatio et ista duo sint dicere, dictio non est cassanda. Et ita si aliquid est cassandum, et assertio non sit cassanda, cum prior sit et ex ipsa non sequitur inconveniens, restat, ut videtur, quod prolatio non sit cassanda. Et hoc non solet fieri. Solutio. Sicut hec diffinitio '*animal rationale*' removetur a quolibet alio ab homine, tamen partes eius non removentur, similiter *assertio cum prolatione* [non] removetur ab hoc enuntiabili, tamen partes eius non removentur. Unde assertio non est cassanda simpliciter, sed habito respectu ad prolationem. Unde neutra est cassanda simpliciter, sed una respectu alterius." The deletion is De Rijk's. Note that although the author implicitly appeals to the Aristotelian distinction between *simpliciter/secundum quid* at the end of this passage, it is not the primary basis for his solution.

[92] Apparently regarded here as a kind of mental act.

[93] Since there are animals and rational things (angels) other than men.

[94] See Roure, *op. cit.*, p. 250, par. 2.01: "Item hec oppinio manifeste contradicit sensui non decepto." ("Again, this opinion clearly contradicts sensation that is not deceived.")

[95] *Ibid.*, p. 269, para. 2.03: "Praeterea ipsi negant sensum: quia ipsi possunt audire Sortem dicere se dicere falsum, ergo Sortem dicere falsum potest dici a Sorte." ("Moreover, they deny sensation. For they are able to hear Socrates say that he is speaking a falsehood; therefore, that Socrates is speaking a falsehood can be said by Socrates.")

[96] *Ibid.*, p. 295, para. 5.05: "Sed isti contradicunt sensui et intellectui." ("But they contradict sensation and understanding.")

batim by Ralph Strode in the late fourteenth century[97] and by Paul of Venice's *Logica magna* in the early fifteenth century,[98] this "refutation" came to be a standard one. It should be added that Richard of Sherwood (item 7 on our list) also rejects *cassatio*, although I have not yet been able to study his text enough to be able to say whether he uses the same arguments.

Nevertheless, although people continued to argue against *cassatio* well into the fifteenth century, they were arguing against straw men. The view died out, as far as I am able to tell, after about 1225—although, curiously, it seems to have been revived in the "post-mediaeval" period by a certain David Derodon and perhaps by others.[99]

VIII

These are the main theories of *insolubilia* during the early period of the literature. In section VII above, I mentioned Bradwardine's survey of previous opinions at the beginning of his own treatise.[100] That survey provides an interesting summary of the early literature. Bradwardine distinguishes eight views in addition to the "Aristotelian" one that he accepts, involving a rather strained appeal to the fallacy *secundum quid et simpliciter*. The eight are: (i) strong *restrictio*; (ii) what appears to be weak *restrictio*, although Bradwardine casts it in the form of the Aristotelian fallacy of "figure of speech";[101] (iii) an appeal to the Aristotelian fallacy of "false cause"—I know of no one who took that approach; (iv) *transcasus*, which we have seen; (v) and (vi), two versions of *cassatio*. Opinion (v) says that no one can say that he speaks falsely or understand that he understands falsely, and so on. This, Bradwardine says, "denies the senses and understanding."[102] Opinion (vi) says that one who nevertheless does say such a thing "says nothing". Bradwardine replies that such a speaker says at least

[97] See Spade, *The Mediaeval Liar*, item liii, p. 87.

[98] *Ibid.*, item l, p. 82.

[99] See Ashworth, *Language and Logic in the Post-Medieval Period*, p. 115; and Paul Vincent Spade, review of Ashworth, *Language and Logic in the Post-Medieval Period*, in: *Dialogue*, 15 (1976), pp. 333-340 at p. 338.

[100] Roure, *op. cit.*, pp. 286-296, para. 2.00-5.08.

[101] The fallacy of figure of speech was regularly appealed to to explain certain fallacies arising in connection with the theory of supposition. See De Rijk, *Logica Modernorum*, vol. II, Part 1, pp. 499-512, especially pp. 511-512. No doubt this is the basis for Bradwardine's classification, although in fact I know of no author who explicitly appeals to the fallacy of figure of speech in connection with *insolubilia*.

[102] See n. 96 above.

words, and so not nothing. In view of the distinctions drawn in the passage quoted above from item 1 in our list, this refutation too seems to miss the mark. It is not clear why Bradwardine treats *cassatio* as two separate views. Burley uses the same characterizations as Bradwardine does, but he treats them as characterizing a single view.[103]

Opinion (vii) in Bradwardine's survey denies bivalence; insolubles are neither true nor false, but have some intermediate value. I know of no one who actually maintained this view in general, although Roger Swyneshed after Bradwardine allowed it to happen in certain special cases.[104] Other authors discuss such views too, even though they do not accept them. William Heytesbury, for instance, writing after Bradwardine, describes and rejects such a theory based on the view that *insolubilia* fail to be "propositions".[105] There may be some connection between the view Heytesbury describes and the theory of *cassatio*, but that is uncertain. In view of the fact that opinions (v) and (vi) in Bradwardine's list are forms of *cassatio*, it is perhaps too much to suppose that opinion (vii) is meant to be yet another form of it. Again, Richard Brinkley in the late fourteenth century describes in some detail and rejects a view that denies bivalence on the basis of a "causal" theory of truth.[106] I know of no one who held such a view.

Finally, opinion (viii) in Bradwardine's survey of previous views is the theory of exercised and signified acts, a theory held by Scotus but apparently by others as well.

Bradwardine's list, with its occasionally obscure features and unidentified views, indicates that we have by no means yet exhausted the early period in the mediaeval *insolubilia*-literature.

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[103] Roure, *op. cit.*, p. 269, para. 2.02-2.03.

[104] See Spade, *Roger Swyneshed's* Insolubilia, p. 180, para. 2. For difficulties in interpreting Swyneshed's theory on this point, see Spade, *Roger Swyneshed's Theory of Insolubilia.*

[105] See Heytesbury, *op. cit.*, p. 19, para. 6. A "proposition" was defined by Boethuis as "an expression signifying a truth or a falsehood". See Boethius, *De topicis differentiis*, PL 64, col. 1174B: "Propositio est oratio verum falsumve significans."

[106] See Spade, *The Mediaeval Liar*, item x, p. 33, fourth previous opinion.